



## International Symposium on Safety Science and Engineering in China, 2012 (ISSSE-2012)

# Avalanche Dynamics in Co-evolutionary Megalopolis Communities' networks

Li Wei<sup>a,\*</sup>

<sup>a</sup> College of Safety and Environment Engineering, Capital University of Economics and Business, Beijing 100070, China

### Abstract

In this article, the stability of the equilibria against perturbations on co-evolutionary megalopolis communities' networks was studied and a number of new features were found. Power law distributions of the differences of the cooperative communities' number after and before perturbations are observed with power exponents changed for different intensities of temptation to defection.

© 2012 Published by Elsevier Ltd. Selection and/or peer-review under responsibility of the Capital University of Economics and Business, China Academy of Safety Science and Technology. Open access under [CC BY-NC-ND license](https://creativecommons.org/licenses/by-nc-nd/4.0/).

**Keywords:** Avalanche, Distribution laws, Co-evolution, Communities' networks

### 1. Introduction

Megalopolis is composed of a large number of communities with simple interactions giving rise to complex global structures and dynamics. The theory of evolutionary games provides a suitable framework for modeling communities' interactions whereas spatial structures have to be taken into account to tackle the essence of co-evolutionary dynamics of real megalopolis. A simple model system is given by Prisoner's Dilemma (PD) game with co-evolutionary dynamics. The PD game is probably the most prominent example of a basic model for explaining the emergence of cooperative behavior in social communities and other complex systems [1-4]. In the PD game communities, who are either cooperative,  $C$ , or defective,  $D$ , act according to their strategies whenever two of them interact. If both communities choose  $C$ , both get a payoff  $R$ ; if both choose  $D$ , both get  $P$ ; if one defective while the other cooperative,  $D$  gets  $T$ , while  $C$  gets  $S$ , where  $T > R > P > S$ . Here, we adopt the values  $R=1$ ,  $T=b>1$ ,  $S=P=0$ . Parameter  $b$  characterizes the temptation to defection against cooperation. The properties of the PD game defined on complex networks exhibit a strong dependence on the static topology[5,6], and recently there are some works studying the interplay of dynamic games and topological structures[7-9]. Remarkably, the interplay of game dynamics and topology can drive the network to self-organized states that cannot be inferred by studying the dynamical processes on static networks.

One essential property of evolutionary games is given by the equilibria or the evolutionary stable states. All stationary states of the game are Nash equilibria [1]. An interesting question is the stability of these equilibria against perturbations, and this problem has been investigated in static networks [6]. How about the avalanche dynamics of the states under the interplay of PD game dynamics and topology? It is a new, fundamental and almost unexplored problem. This is the central

\* Corresponding author. Tel.: +86-010-83951020; fax: +86-010--83951020.

E-mail address: [liwei@cueb.edu.cn](mailto:liwei@cueb.edu.cn)

task to study the avalanche dynamics of megalopolis communities' networks under slight perturbations by considering co-evolutionary dynamics.

## 2. Model

The avalanche dynamics of megalopolis communities' networks is studied on the basis of the network structure proposed in our previous work [7]. The main feature of the model is that the network has both fixed local links and adjustable long-range links. In the network, all nodes, representing communities in the game, are distributed in a two-dimensional lattice. All communities have nearest-neighbor interactions by  $n$  local links without direction ( $n = 4, 3, 2$  for internal, edge and corner nodes, respectively). These local interactions are assumed to be fixed during the whole process of the game. On the other hand, all communities have limited needing to establish long-range links (for simplicity, we take a single long-range links for every communities). All long-range links are directed by arrows. For a given node a long-range links is called active if the arrow is from the node while passive if the arrow is toward the node. In the network, nodes play PD game with all nodes connected (including all nearest neighbors, one active long-range link's neighbor and all passive long-range links' neighbors), following the role: *learning from the best* to adjust their strategies and long-range links. Each communities (say A) compares the payoff of himself with those of all his connected neighbors in the previous round and finds out the community with the best payoff (or chooses one with equal probabilities if there are multiple best neighbors). If A has the largest payoff, A keeps its own previous strategy and long-range link; otherwise, A learns from the best neighbor (say B) by simultaneously taking the same strategy of B and rewiring its active long-range link to B's active long-range link neighbor (if the new long-range link does not achieve more profit, keeps the old links) with probability  $W = 1/(1 + e^{-\beta \Delta P})$ , where  $\Delta P$  denotes the remainder to take the community's total payoff from its best neighbor's, and  $\beta$  characterizes the noise effects (here, we choose  $\beta = 0.1$  for numerical simulations). In this work, we will study the influences of perturbations on the steady states and the characteristic behaviors of avalanches induced by the perturbations.

## 3. Numerical simulation

We will study the dynamics of avalanches as follows. After the co-evolutionary megalopolis communities' network has reached a stationary state, and adverse strategy is assigned to a community randomly chosen. The insertion of a new strategy offers new opportunities for learning to the perturbed community itself and to its nearest neighbors and again neighbors of the nearest neighbors and so on. A perturbation can thus lead to an avalanche of strategy and interaction changes until a new stationary state is reached again. We defined a quantity to characterize the avalanche dynamics, which is  $\Delta N_C$ , the difference between the numbers of cooperators after and before the perturbation. All simulations results for different  $b$ 's are taken from 2000 different realizations of initial linking and strategy conditions. We observe distinct regimes of the avalanche dynamics, depending on the control parameter  $b$ .

For weak-temptation,  $1 < b < 1.2$ , critical behavior occurs, i.e., the probability distributions of cooperator change  $P(|\Delta N_C|)$  show clearly power laws of  $P(|\Delta N_C|) \propto |\Delta N_C|^{-\alpha}$  with scaling exponent  $\alpha = 2.42 \pm 0.10$  (see Fig. 1). In this regime, small perturbations lead only to small avalanches which are necessary to reestablish stationary equilibria.

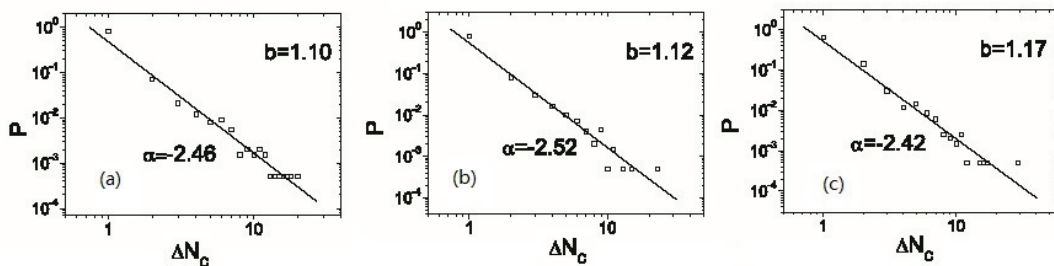


Fig. 1. Probability distribution  $P(|\Delta N_C|)$  for weak temptation. Open squares show numerical data. The black line shows fitting line of these data.

The avalanche behaviors for large  $b$  become much more complex as well as interesting. First, for  $b \geq 1.2$ , one can observe power law distribution of  $P(|\Delta N_C|)$  while with different power exponents  $P(|\Delta N_C|) \propto |\Delta N_C|^{-\alpha}$ . In Fig. 2 find  $\alpha \approx 3$  in a large of  $b$ , but  $\alpha \approx 2$  (see Fig. 2) for certain particular  $b$  values. The difference of the scaling exponent  $\alpha$  may be due to the transitions of the asymptotic network structures. In [7] we found that the stable model's structure change from single-scale to broad-scale, from broad-scale to scale-free, and from scale-free to all-defector states at  $b = 2.0$ ,  $b = 2.5$ , and  $b = 2.0$  respectively, and these may be the reasons why we observe  $\alpha \approx 2$  at these particular values, sharply different from  $\alpha \approx 3$  in a wide regions of  $b \geq 1.2$ .

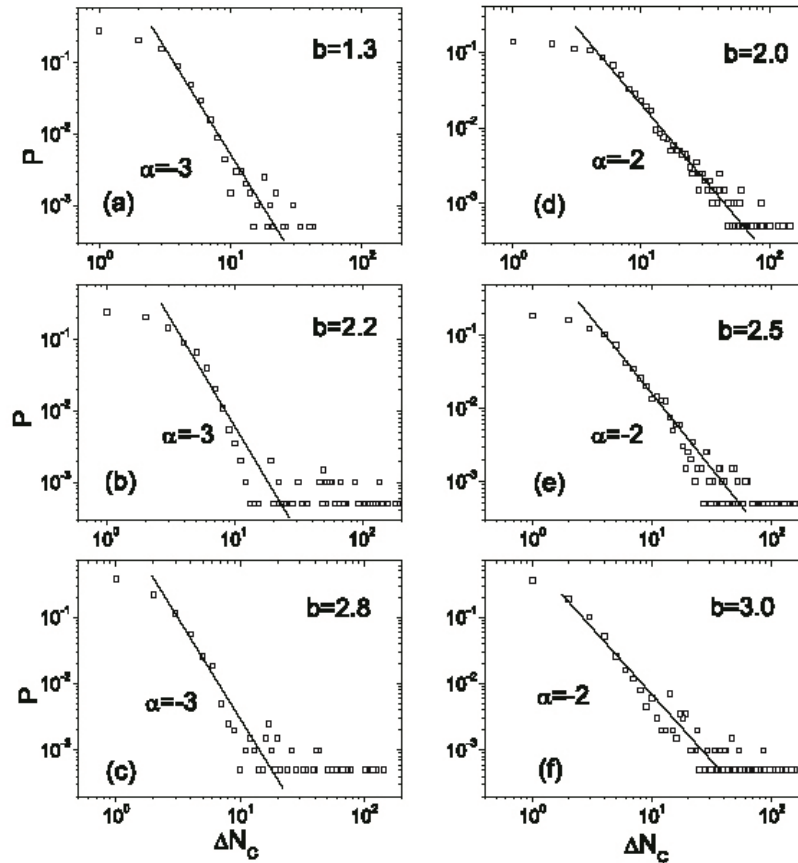


Fig. 2. Probability distribution  $P(|\Delta N_C|)$  for  $b \geq 1.2$ . Open squares show numerical data.

For a comparison, we study also the avalanche dynamics of the static megalopolis communities' network (the network exactly the same as our model except all long-range links fixed or static). In Fig. 3 we show the results. For  $b < 1.2$  probability distributions  $P(|\Delta N_C|)$  follow also power laws  $P(|\Delta N_C|) \propto |\Delta N_C|^{-\alpha_F}$  with different scaling exponent  $\alpha_F = 1.86 \pm 0.10$ . The exponents of static network are smaller than that of co-evolutionary network, because adjustable long-range links can help to establish effectively new balance after the perturbation, and thus finish the avalanche processes in much fewer steps. For  $b \geq 1.2$ , we can no longer observe same behaviors in static network as co-evolutionary network. In static network, cooperators form clusters owing to the random long-range links when  $b$  is large, and cooperator clusters are separated by defectors and the percentage of cooperators is lower than its in co-evolutionary networks. In static networks, when the perturbation happened in the inner of defector-sea the perturbation disappears rapidly. On the contrary, if the perturbation happened in the boundary of defector-sea and cooperator clusters or happened in the inner of cooperator clusters, the perturbation will be transported through small cooperator clusters and the thin "cha

of the defector-sea. Though the perturbation can be transported in large-scale in static networks, the effect of it is limited owing to the fixed long-range links.

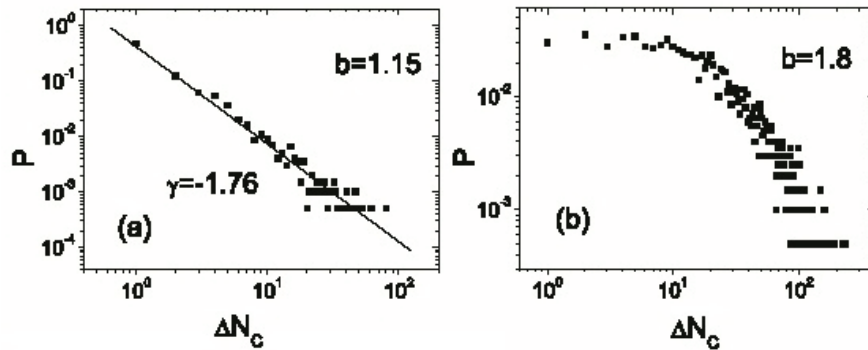


Fig. 3. Probability distribution  $P(|\Delta N_c|)$  for fixed megalopolis communities' networks.

#### 4. Conclusion

In conclusion, we have studied the avalanche dynamics of the co-evolutionary megalopolis communities' network under the PD game law and *learning from the best* interactions. Rich phenomena can be observed in the avalanche dynamics. In particular, we find by numerical simulations that the probability distribution of the average absolute change of the number of cooperators before and after random perturbations shows power law with different scaling exponents for different intensities of temptation to defect. These interesting phenomena do not exist in static megalopolis communities' network and the diversity of avalanche dynamics in the co-evolutionary networks may come from the interactions between communities capable to adjust their long-range links owing to the learning ability. Our results represent a first step in the unexplored domain of the properties with generic self-organized coupling between dynamics and topology of co-evolutionary networks. We will go further in the future to explore new features in these co-evolution processes and to understand the mechanisms underlying these features.

#### Acknowledgements

This work is supported by National Natural Science Foundation of China under Grant No. 11005075.

#### References

- [1] J. M. Smith, 1982. *Evolution and the Theory of Games*, Cambridge University Press, Cambridge, England.
- [2] M. A. Nowak, R. M. May, *Nature* 359(1992) 826.
- [3] C. Hauert, S. De Monte, J. Hofbauer, K. Sigmund, *Science* 296(2002) 1129.
- [4] M. Nowak, K. Sigmund, *Nature* 437 (2005) 1291.
- [5] P. Holme et al, *Phys. Rev. E* 68 (2003) 030901(R).
- [6] H. Ebel, S. Bornholdt, *Phys. Rev. E* 66 (2002) 056118.
- [7] W. Li, X. Zhang, G. Hu, *Phys. Rev. E* 76 (2007) 045102 (R).
- [8] M. G. Zimmermann et al, *Phys. Rev. E* 69 (2004) 065102 (R).
- [9] P. Holme and G. Ghoshal, *Phys. Rev. Lett.* 96 (2006) 098701.